Performance Based Learning and Assessment Task

Choosing the Best Cylinders

I. **ASSESSSMENT TASK OVERVIEW & PURPOSE:**
In this task, students will discover and justify the effects of changing dimensions on cylinders and other three-dimensional figures.

II. **UNIT AUTHOR:**
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III. **COURSE:**
Geometry

IV. **CONTENT STRAND:**
Geometry

V. **OBJECTIVES:**
Students will be able to describe the effects of changing dimensions on a three-dimensional figure and determine the relationship between a change in one dimension and the surface area or volume.

VI. **REFERENCE/RESOURCE MATERIALS:**

VII. **PRIMARY ASSESSMENT STRATEGIES:**
Students will communicate reasoning/justification of the following:

-effect of increasing the radius on the volume and surface area of a cylinder compared to increasing the height of the cylinder by the same percent increase.

-proportional relationship between the new volume by determining a fair price for the new product

-relationship between changing dimensions for similar figures and the resulting effects on surface area and volume.

VIII. **EVALUATION CRITERIA:**
For each question, students should provide a mathematical justification for their solution.

IX. **INSTRUCTIONAL TIME:**
30 minutes

Choosing the Best Cylinders for Customers

Strand
Geometry

Mathematical Objective(s)
Students will describe the effects of changing dimensions of three-dimensional objects on the volume and surface area.

Related SOL

G.13 The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

G. 14 The student will use similar geometric objects in two- or three-dimensions to

b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;
c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and
d) solve real-world problems about similar geometric objects.

NCTM Standards: The student will

- **Geometry 9-12:** explore relationships among classes of three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them
- **Algebra 9-12:** Use mathematical models to represent and understand quantitative relationships
- **Reasoning and Proof:** make and investigate mathematical conjectures; develop and evaluate mathematical arguments and proofs
- **Communication:** communicate their mathematical thinking coherently and clearly to peers, teachers, and others.

Additional Objectives for Student Learning (include if relevant; may not be math-related):

Materials/Resources

Assumption of Prior Knowledge

- Students should be able to solve for volume and surface area of three-dimensional figures
- Students should be able to use proportional reasoning to determine relationships between volumes and surface areas.
- Students should be able to calculate changing dimensions by percent increases or decreases.

Introduction: Setting Up the Mathematical Task

In this task, the students will explore the effects of changing dimensions on the volume and surface area of cylinders and other three-dimensional figures. Students will discover the proportional relationship when comparing similar figures. Students will also compare the effect of changing a radius of a cylinder versus changing the height by the same percentage.

In order to introduce this task, the teacher could show two cylinders—one short but wide and another tall but narrow but with approximately the same volume. If these are constructed out of paper, the following dimensions will work:

Short Cylinder: circumference of 19 units, and height of 6 units

Tall Cylinder: circumference of 14.5 units, and height of 10 units

Ask the students which one holds more liquid and have the students vote. Is there an overwhelming response towards one of the cylinders? Have the students compute the volumes. Did they expect the volumes to be that close? (Note: more precise measurements can result in exactly the same volume and may be done if desired.)

Student Exploration

In choosing what size cup to serve beverages, people tend to prefer tall, thin containers, as this appears to hold more liquid. Is this really the case?

Question 1:

a. Consider a cylindrical cup that has a radius of 2 inches and a height of 6 inches. To improve customer satisfaction, a new cup is used that has decreased the radius by 25% but increased the height by 25%. Is this a better deal for the customer?

b. In part a, the radius decreases by the amount that the height increased. Is the result what you expected? Why or why not? Explain why this result occurred.

c. If the current cup costs $1.50, what would a fair price for the new cup be?

d. Which would increase the volume more: doubling the radius or doubling the height? Why?

Question 2:

A movie theatre wants to charge a fair price for the three sizes of cylindrical containers of popcorn.

Small: radius of 1.5 inches and a height of 6 inches

Medium: radius is double the size of the small, same height as the small

Large: radius is double the size of the small, and double the height of the small

a) If a small container costs $1.50, what should the prices of the medium and large containers be?

b) What is the relationship between the small dimensions and the large dimensions? What is the relationship between the small and the medium dimensions? Which pair(s) is/are similar?

c) How do the volumes compare between the small, medium, and the large dimensions? Explain mathematically why this relationship occurs.

Monitoring Student Responses

- Ensure that students are showing all work in drawing conclusions. Also, they should be comparing the proportions of the new volumes/surface areas in order to discover the similar figure relationships on changing dimensions.

Assessment List and Benchmarks

- Extension Questions
  - **Questions**
    1. Why does changing the radius affect the volume more than changing the height by the same percentage?
    2. What is the effect on the volume of doubling only the height of a cylinder?
    3. How much should you change the height of the cup to increase the volume by the same amount as doubling the radius?
  - **Journal/writing prompts**
    - If you were to consider the effects of changing the radius and height on the amount of paper needed to make the cup, does doubling the radius change the surface area the same amount as doubling the height? Why or why not?
    - In the task, we considered the effects of changing dimensions for cylinders. Will the same results occur for cones? What happens when you change one, two, or all three of the dimensions of rectangular prisms when calculating the volume?
- Students should be assessed on the following: (5 point scale)
  - Correct quantitative solutions (2 points)
  - Justification/work provided to support all solutions (3 points)

- Solutions:

Question 1:

A) Old volume = $\pi \cdot 2^2 \cdot 6 = 75.36 \text{ in}^3$

New volume = $\pi \cdot 1.5^2 \cdot 7.5 = 52.99 \text{ in}^3$

Since the new, taller cup has less volume, it is not a better deal for the customer.

B) Since the radius is squared in calculating the volume, it has a bigger effect on the volume than the height.

C) $\frac{75.36}{52.99} = \frac{1.50}{x}$

x = $1.05

Question 2:

A) Small : $\pi \cdot 1.5^2 \cdot 6 = 42.39 \text{ in}^3$

Medium: $\pi \cdot 3^2 \cdot 6 = 169.56 \text{ in}^3$

Large: $\pi \cdot 3^2 \cdot 12 = 339.12 \text{ in}^3$

Price of Medium:

$$\frac{42.39}{169.56} = \frac{1.50}{x}$$
$$x = \$6$$

Price of Large:

$$\frac{42.39}{339.12} = \frac{1.50}{x}$$
$$x = \$12$$

B) In the medium container, only the radius is doubled. In the large container, both the radius and the height are doubled. The small and the large containers are similar since all of the dimensions have been changed by the same proportions.

C) The volume of the medium container is 4 times as large. Since the radius is squared, the effect on the volume is 2^2= 4 times. The volume of the large container is 8 times as large. This occurs because of changing the radius and the height results in $2^2 \cdot 2 = 8$ times.